ΔT_{ad} = adiabatic temperature rise: $\frac{H_r C_o}{\rho C_p}$

Δp = pressure drop over reactor length

Δp_o = initial pressure drop = $\frac{8Q\eta_o L}{\pi R_o^4}$

Δp^* = dimensionless pressure drop: $\Delta p / \Delta p_o$

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Optimum Dimensions for Pipeline Mixing at a *T*-Junction

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Conditions are identified which insure optimum pipeline mixing at a *T*-junction. Analytical expressions, that result from detailed measurements of both near and far-field momentum-dominated jet trajectories in a crossflow, are renormalized in terms of pipe coordinates. It is demonstrated that a simple scaling law exists for the limiting geometry of either large or small jet-to-pipe diameter ratios. The scaling laws are shown to accurately correlate existing data for each limiting geometry.

SCOPE

Turbulence promotes most important chemical reactions, heat transfer operations and mixing and combustion processes in industry. Effective use of turbulence increases reactant contact and decreases reaction times which can significantly reduce the cost of producing many chemicals. It is common in existing chemical process units to mix two fluids at a tee in a pipe with subsequent transport to other locations. For numerous continuous pipeline mixing applications, the feed jets are directed perpendicular to the pipe axis. The distance necessary to achieve a desired degree of uniformity of concentration or temperature in the pipe depends on the following quantities: ratio of jet-to-pipe diameter, uniformity criterion, ratio of jet-to-pipe velocity, ratio of specific gravities of the two feed streams and the pipe or jet Reynolds number and surface roughness. Simpson (1975) and Gray (1982) have prepared reviews of pipe mixing with tees and other geometries.

Chilton and Genereaux (1930) recorded the pioneering, although elementary, experimental results for optimum mixing with a *T*-junction. Most of the existing data were taken by Forney and Kwon (1979) using improved quantitative techniques for a range of diameter ratios $0.025 \leq d/D \leq 0.2$. Forney and Kwon also developed an expression to predict optimum dimensions for a side-tee mixer by assuming that optimum mixing conditions were achieved if the jet and pipe axis coincided at a fixed distance from the point of injection. Although the constraint of geometric similarity imposed on their near-field jet trajectories within a pipe provided useful results, the slope of their theory was imperfect compared with existing data for large diameter ratios $d/D > 0.035$. Moreover, the form of

their assumed jet trajectory lacked validity for smaller diameter ratios, where little data existed.

The present paper records additional data for optimum mixing at a *T*-junction for small diameter ratios. Existing published data including the recent results of Maruyama et al. (1981) are also introduced. In addition, the results of recent detailed measurements of jet trajectories by Wright (1977) are used to extend the concept of optimum mixing proposed by Forney and Kwon. As a result, considerable improvement in data correlation is demonstrated for all useful mixer dimensions.

CONCLUSIONS AND SIGNIFICANCE

Two simple scaling laws are derived which assure optimum pipeline mixing of constant density fluids at a *T*-junction. Recent detailed measurements of near and far-field momentum dominated jet trajectories have been adapted to pipeline jet injection. The constraint of geometric similarity imposed upon these jet trajectories within the pipe assumes that optimum mixing is achieved if the jet and pipe axis coincide at a fixed distance from the point of injection. With the proper choice of boundary conditions, excellent agreement with existing data has been demonstrated with the following relationships between the jet-to-pipe diameter and velocity ratio

$$\frac{d}{D} = \frac{0.27}{Rf^2} \quad \text{for } R < 6$$

and

$$\frac{d}{D} = \frac{0.28}{Rg^{3/2}} \quad \text{for } R > 6$$

where $f(R) = 0.17/(0.1 + 0.35/R^{1.25})$ and $g(R) = 0.83 + 0.2\ lnR$. These expressions are shown to be independent of pipe Reynolds number and to provide optimum mixing provided the measurement position is at least two pipe diameters from the injection point. For jet Reynolds numbers $Re_j < 9 \times 10^3$, the predicted optimum velocity ratio must be computed from the expression

$$R_j/R = (Re_j/9 \times 10^3)^{1/2}$$

where the value of R is determined from the expressions given above.

INTRODUCTION

The first systematic study of pipe mixing with a *T*-junction was conducted by Chilton and Genereaux (1930) who used smoke visualization techniques to determine optimum mixing conditions at a glass tee. They concluded that right angle configurations were as effective as any other geometry for good mixing. Chilton and Genereaux also found that when the ratio of the velocity of the secondary flow to the velocity of the main flow was 2–3 for their geometries, satisfactory mixing was obtained in 2–3 pipe diameters. Narayan (1971) used quantitative methods to measure the degree of mixing of air-carbon dioxide feed streams in three pipeline mixers. Narayan, like Chilton and Genereaux, found it was possible to achieve quality mixing in a few diameters with perpendicular jet injection devices but that parallel flow devices required up to 250 pipe diameters. Ger and Holley (1974) compared standard deviations from measured tracer concentration distributions far downstream (20–120 pipe diameters) from a *T*-junction geometry. These results were compared to other mixer geometries with tracer injection points near the wall or at the center of the pipe. For a limited range of jet-to-pipe diameter ratios, Ger and Holley also concluded that complete mixing could be achieved in a shorter pipe length with right angle configurations.

More recently, Forney (Winter, 1975; Forney and Kwon, 1979; Lee, 1981) and Maruyama, Suzuki and Mizushina (1981) independently studied the right angle injection of fluid into a pipeline. Both studies concluded that the ratio of jet-to-pipe diameter versus jet-to-pipe velocity for optimum mixing was independent of measurement position down the axis of the pipe provided the measurements were made at 2 or more pipe diameters downstream from the point of injection. They also concluded that the mixing criterion was independent of the pipe Reynolds number. The criterion for optimum mixing used by Forney and Kwon was to center the jet and pipe axis at measurement points between 2 and 5 pipe diameters. Although their mixing criterion was different than that of Ger and Holley and Maruyama et al., the measured results of all three studies were identical within experimental scatter of the data. Forney and Kwon also demonstrated that the jet Reynolds number could influence the jet-to-pipe velocity ratio to achieve optimum mixing if $Re_j < 9 \times 10^3$.

The problem of predicting the optimum dimensions for jet injection in a pipe is a difficult one. The degree of mixing downstream from the point of injection is dominated by the properties of the jet near the point of injection. Near the inlet the jet entrains ambient pipe fluid and bends over in the crossflow. Since this formally constitutes a three-dimensional turbulent shear flow embedded in a fully developed turbulent pipe flow, it is unlikely that full analytical solutions are possible.

The experiments and analysis of Forney (Winter, 1975; Forney and Kwon, 1979) suggested a physical picture, although incomplete, which provided an elementary understanding of the process and led to the development of a useful scaling law. The critical assumption in their work was that one could achieve rapid unretarded mixing of the jet and pipe fluids by centering the jet in the tube at a fixed distance x_o (ideally the Lagrangian integral length scale ℓ_L) down the pipe axis from the point of injection, $x = 0$. This criterion provided geometrically similar jet trajectories within the tube. The reasoning behind their approach was that over the distance $o \leq x \leq x_o$ jet induced turbulence dominates the mixing process while ambient pipe turbulence dominates the mixing for $x > x_o$. Similar arguments for the mixing of a passive tracer within turbulent pipe flows were made by Kalinske and Pien (1944).

In the present paper the experimental results of Lee (1981) are presented for conditions of optimum pipe mixing at a *T*-junction. In particular, the experiments of Lee were taken over a range of small diameter ratios $0.008 < d/D < 0.035$ where little data previously existed. Recent additional experimental results of Maruyama et al. (1981) along with other available data are also included in the first complete correlation of optimum jet-to-pipe velocity and flow ratio versus the jet-to-pipe diameter ratio for optimum mixing conditions at a *T*-junction. The theoretical model of Forney and Kwon (1979) has been extended with the adaptation of recent detailed measurements of jet trajectories in a crossflow by Wright

(1977). These results are found to support arguments that the optimum dimensions for pipe mixing at a T-junction can be predicted by using one of two simple scaling laws depending on the magnitude of the jet-to-pipe diameter ratio. The results are also used to predict and correlate measurements of the decay in concentration along the jet centerline.

SUMMARY OF PREVIOUS WORK

Various mixing criteria have been used by experimentalists to determine conditions of optimum mixing at a T-junction of the type shown in Figure 1. Using either air or water as working fluids, the investigators have typically injected a chemical tracer into the fluid medium, measured the distribution of the tracer downstream from the point of injection and compared the measurements to a given mixing criterion. Table 1 summarizes all of the previous experimental work including the range of parameters and mixing criteria used.

Attempts to predict velocity, diameter and flow ratios for optimum mixing conditions at a T-junction are limited to two empirical expressions developed by Ger and Holley (1974) and Maruyama et al. (1981), which are applicable over a limited range of operating conditions. In addition, the theoretical development of Forney and Kwon (1979) exists which appears to be useful for most anticipated geometries. These expressions are listed in Table 2, where the optimum diameter ratio d/D is related to the velocity ratio $R(= u_o/v)$ and the optimum flow ratio $q/Q(= d/D)^2R)$ is expressed in terms of the diameter ratio.

It can be noted from the expressions in Table 2 that the theory of Forney and Kwon yields the values $q/Q = 0.775(d/D)^{1.5}$ for $d/D \rightarrow 1.0$ and $q/Q = 0.11\ (d/D)$ for $d/D \rightarrow 0$. These expressions compare favorably with the empirical results of $q/Q = 1.49\ (d/D)^{1.58}$ for $0.03 < d/D < 1.0$ by Maruyama et al. and $q/Q = 0.125\ (d/D)$ for $d/D < 0.02$ by Ger and Holley. If the Reynolds number of the jet $Re_j < 9 \times 10^3$, the optimum velocity and flow ratios are computed as R_j and $(q/Q)_j$ in the theory of Forney and Kwon.

THEORY

The nature of the trajectories of round turbulent, non-buoyant jets in a crossflow has received considerable attention. As demonstrated recently in a detailed study and review of the problem by Wright (1977), dimensional arguments and detailed experimental data suggest scaling laws for the jet trajectory of the following form:

$$\frac{z}{l_m} = f(R)(x/l_m)^{1/2}, \qquad x < x_c \tag{1}$$

and

$$\frac{z}{l_m} = g(R)(x/l_m)^{1/3}, \qquad x > x_c. \tag{2}$$

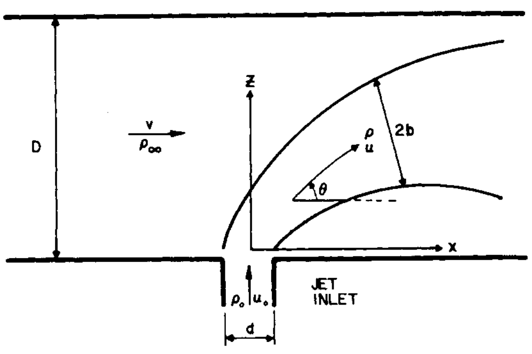

Figure 1. Coordinate system for jet injection at a T-junction.

TABLE 1. SUMMARY OF EXPERIMENTS

Jet Fluid	Pipe Fluid	Jet Diameter (cm)	Pipe Diameter (cm)	$R\ (u_o/v)$	Pipe Re	Jet Re_j	Measurement Point in Pipe Diameters	Measured Variable	Mixing Criterion	Reference
Air & $TiCl_4$	Air	0.68–2	4.45	1.5–3	4.3×10^3 -1.8×10^4	$>1.8 \times 10^4$	2–3	Visual smoke conc.	Visual smoke uniformity	Chilton and Genereaux 930)
Aq. 0.5N HNO_3	Aq. 0.5N NaOH	0.635	0.635	1	$1. \times 10^4$ $-4. \times 10^4$	$1. \times 10^4$ $-4. \times 10^4$	6–7	Temp.	97% of final temp. rise	Swanson (1958)
Aq. NaCl	Water	0.08–0.32	15.24	6–24	$6. \times 10^4$	7.5×10^3	20–120	Elec. conductivity	Conc. stand. deviation	Ger and Holley (1974)
Air & $TiCl_4$	Air	0.42–1.5	5.0	1.5–3.3	4×10^3 -2×10^4	$>1. \times 10^4$	2–3	Visual smoke conc.	Visual smoke uniformity	Winter (1975)
Air & 1% CH_4	Air	0.16	6.35	2–7	$2. \times 10^3$ $-9. \times 10^4$	5×10^2 -2.5×10^4	2–5	CH_4 conc.	Max. conc. centered on pipe axis	Forney & Kwon (1979)
Air 25°C	Air ~35°C	0.5–1.3	5.1	3–4	1.6×10^4 -6.3×10^4	8.2×10^3 -2.3×10^4	2–10	Temp.	Temp. stand. deviation	Maruyama Suzuki & Mizushina (1981)
Air 19% CO_2	Air	1.58	5.25	2.7	4.6×10^4	3.74×10^4	10	CO_2 conc.	Equal CO_2 conc. at pipe axis & periphery	Reed & Narayan (1979)
Air & 0.3% CH_4	Air	0.1–1.27	11.43	2.9–28.3	1.3×10^4 -3.2×10^4	1.1×10^3 -7.2×10^3	2–10	CH_4 conc.	Max. conc. centered on pipe axis	Lee (1981)

TABLE 2. PREDICTION OF OPTIMUM GEOMETRY

Equation	Author	Comments
$d/D = \frac{\alpha}{R} + \frac{\beta}{R^2}$	Forney and Kwon (1979)	good for all Re_j. R, $d/D < 1.0$
$q/Q = \frac{\alpha}{2}\left(\frac{d}{D}\right)\left\{1 + \left[1 + \frac{4\beta}{\alpha^2}\left(\frac{d}{D}\right)\right]^{0.5}\right\}$		
for $\alpha = 0.11, \beta = 0.6$		
if $Re_j < 9 \times 10^3$		
$R_j/R = \left(\frac{Re_j}{9 \times 10^3}\right)^{1/2}$		
$(q/Q)_j = (q/Q)\left(\frac{Re_j}{9 \times 10^3}\right)^{1/2}$		
$\frac{d}{D} = \frac{0.125}{R}$	Ger & Holley (1974)	$R > 10.0$, $d/D < 0.01$ $Re_j = 7.5 \times 10^3$
$q/Q = 0.125\,(d/D)$		
$d/D = \frac{2.65}{R^{2.4}}$	Maruyama, Suzuki & Mizushina (1981)	$R < 6.0$ $1.0 > d/D > 0.03$
$q/Q = 1.49\,(d/D)^{1.58}$		$Re_j > 8 \times 10^3$

Here, $R(= u_o/v)$ is the velocity ratio, u_o is the mean velocity at the jet inlet, v is the crossflow velocity, $l_m = dR$ is the momentum length, d is the jet inlet diameter, x is the distance measured in the direction of the crossflow where x_c is the point of intersection of Eqs. 1 and 2 and z is measured perpendicular to the crossflow.

Although approximations to the functions $f(R)$ and $g(R)$ appearing in Eqs. 1 and 2 can be predicted theoretically (Hoult, Fay and Forney, 1969; Hoult and Weil, 1972), we now find it convenient to use the experimental values of Wright shown in Figure 2 to derive the continuous functions,

$$f(R) = \frac{0.17}{0.1 + 0.35/R^{1.25}} \tag{3}$$

and

$$g(R) = 0.83 + 0.2\, lnR. \tag{4}$$

We now seek geometrically similar jet trajectories such that the jet the pipe axes coincide ($z/D = 0.5$) at a fixed distance x_o from the point of injection as was originally proposed by Forney and Kwon. Here we anticipate that $x_o \sim l_L$ the Lagrangian length scale down the tube axis where $l_L/D = 0.4\,Re^{1/8}$. We also assume that jet induced turbulence dominates the mixing of the two fluids over the range $o < x < x_o$ while fully developed pipe turbulence dominates mixing for $x > x_o$.

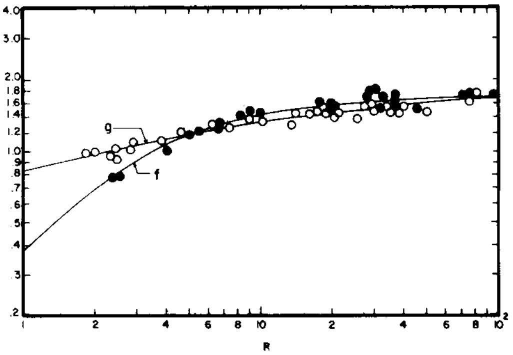

Figure 2. Variation of coefficients for momentum-dominated jet trajectories after Wright (1977): f is near-field coefficient; g is far-field coefficient.

To determine the range of validity of Eqs. 1 and 2, we normalize the coordinates x, z with the pipe diameter D and seek the intersection of both the near and far-field trajectories at constant $z = z_c$. This yields

$$\frac{z_c}{D} = \left(\frac{l_m}{D}\right)^{1/2} f(R)(x_c/D)^{1/2} = \left(\frac{l_m}{D}\right)^{2/3} g(R)(x_c/D)^{1/3} \tag{5}$$

where x_c, z_c represent the coordinates of the beginning of the far-field scaling law Eq. 2. Solving for x_c from Eq. 5 we find

$$\frac{x_c}{D} = \left(\frac{l_m}{D}\right)(g/f)^6. \tag{6}$$

We now assume that $l_m/D \sim 0(1)$ for conditions of optimum mixing and in general $x_o/D \sim l_L/D \sim 0(1)$ for tube Reynolds numbers in the range $10^4 < Re < 10^6$. Thus, the intersection of f and g at $R \simeq 6$ where $f = g$ in Figure 2 implies from Eq. 6 that one must seek geometrically similar trajectories over the range $x < x_o$ with Eq. 1 if $R \lesssim 6$ and Eq. 2 if $R \gtrsim 6$.

Normalizing the coordinates x, z in Eqs. 1 and 2 with respect to duct diameter D, one now determines similar trajectories such that $z/D = 0.5$. This is accomplished by assuming $x_o/D = x_f$ in Eq. 1 for small R and $x_o/D = x_g$ in Eq. 2 for large R where the boundary conditions $x_f \simeq x_g \simeq 0(1)$ are empirically determined constants. One now has

$$\frac{d}{D} = \frac{0.25}{x_f R f^2}, \qquad R < 6 \tag{7}$$

and

$$\frac{d}{D} = \frac{0.35}{{x_g}^{1/2} R g^{3/2}}, \qquad R > 6. \tag{8}$$

Equations 7 and 8 are assumed to represent the diameter ratio for optimum mixing at a T-junction with the boundary conditions x_f and x_g to be determined from experiment. In addition, the optimum flow ratio can be determined from Eqs. 7 and 8 since $q/Q = (d/D)^2R$.

Since dilution and mixing of the fluid from the side tee of a T-junction is dominated by jet induced turbulence for a distance of approximately one pipe diameter from the point of injection, one can determine the rate of decay of a passive tracer near the point of injection. The dilution measurements of Wright (1977) are not as complete as their trajectory measurements. Thus we use the previous results of Forney and Kwon (1979) for small R where the jet trajectory is described by the near-field 1/2 power law Eq. 1, and dimensional arguments for large R where the jet trajectory is described by the far-field 1/3 power law Eq. 2, to predict the decay of the jet centerline concentration.

For small R the analysis of Forney and Kwon yielded the results that c_m/c_o depends only on the dimensionless group $R(x/D)^{1/2}$ where c_m is the jet centerline concentration and c_o is the tracer concentration at the point of injection. For large R we note that the jet trajectory is described by Eq. 2 where

$$\frac{z}{l_m} \sim (x/l_m)^{1/3}. \tag{9}$$

Dimensional arguments also suggest that the local radius of the jet b is proportional to its rise or

$$b \sim z. \tag{10}$$

Writing an expression for the conservation of mass of the passive tracer and substituting from Eqs. 9 and 10 above one has

$$\frac{c}{c_o} = \left(\frac{b_o}{b}\right)^2 R \sim \frac{(l_m/D)^{2/3}}{R(x/D)^{2/3}} \tag{11}$$

where c is the bulk mean concentration of tracer and $c_m \alpha c$. Since l_m/D is approximately a constant for conditions of optimum mixing, we find that the maximum jet concentration decays in the following manner for large R

$$\frac{c_m}{c_o} \sim \frac{1}{R(x/D)^{2/3}} \tag{12}$$

EXPERIMENTAL PROCEDURE

The experimental technique consisted of measuring the centerline position and concentration of a fully turbulent methane jet issuing normally into a turbulent airstream flowing through a 11.43 cm lucite ID tube. The primary airflow was maintained with a blower located downstream from the jet inlet and measured with a pitot tube as shown in the schematic of the apparatus in Figure 3. The air velocity within the larger tube was regulated by restricting the cross sectional area of the entrance which increased the head loss and lowered the flow rate. It was found that tube Reynolds numbers in the range of $1.26 \times 10^4 < \mathrm{Re} < 3.2 \times 10^4$ could be obtained in this manner.

The mean jet inlet velocity u_o was measured with a calibrated rotameter as indicated in Figure 3 and controlled with a high pressure regulator. The jet diameter was varied in the experiment by using one of several lucite and glass capillary inserts covering the range $0.1\ \mathrm{cm} < d < 1.27\ \mathrm{cm}$. The jet airflow was regulated such that a maximum value of the jet Reynolds number $\mathrm{Re}_j = 7.2 \times 10^3$ could be achieved. This was sufficient to center the jet axis in the center of the pipe and to insure fully developed turbulence within the jet.

Jet concentration profiles were determined with one of several probes which are adjustable along the tube radii and were located at a number of positions downstream from the jet inlet. Methane concentrations were measured along the tube diameter with a flame ionization detector (Beckman). The methane concentration of 0.3% in air was regulated from a high pressure tank and the total jet flowrate was measured by a rotameter.

The procedure for a typical run with a fixed jet-to-pipe diameter d/D was to establish the airflow in the large tube at one of three values of the

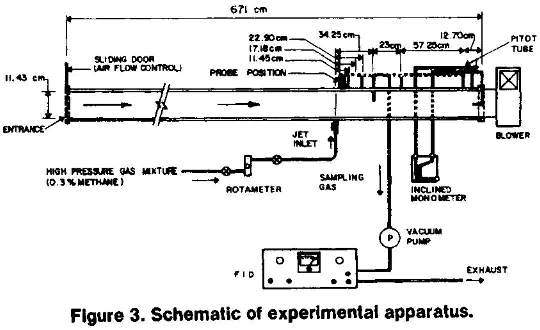

Figure 3. Schematic of experimental apparatus.

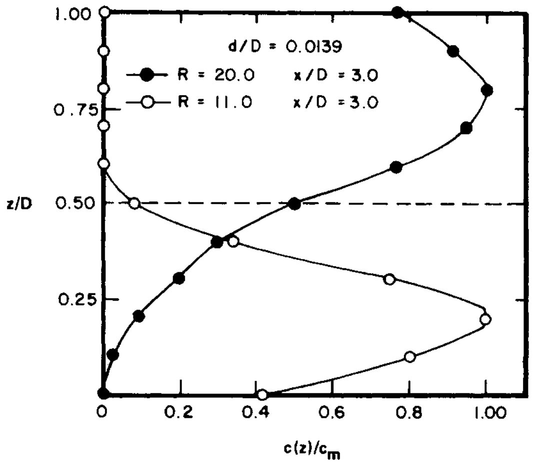

Figure 4. Methane profiles for cases of poor mixing. Solid symbols are overpenetration. Open symbols are underpenetration.

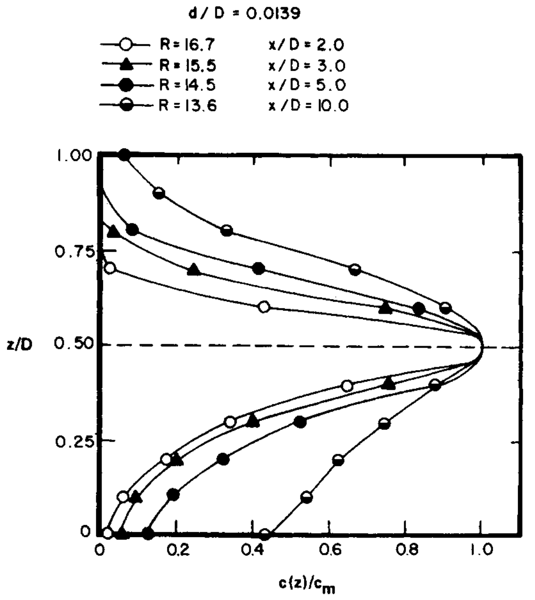

Figure 5. Methane profiles for optimum mixing.

Reynolds number. Concentration profiles were varied and determined at each value of Re for probe positions $x/D = 2.0, 3.0, 5.0$ and 10.0 while increasing the jet velocity until the centerline of the methane jet coincided with the pipe centerline. Two cases of poor mixing in which the jet has under or overpenetrated the pipe centerline are shown in Figure 4. Several concentration profiles which represent optimum mixing are indicated in Fig. 5. The optimum values of the velocity ratio ($R = u_o/v$) for geometrically centered jets at each probe position are shown in Table 3. The decay of methane concentration along the jet centerline was also recorded at several additional probe locations downstream from the jet inlet for jets centered at $x/D = 2.0, 3.0, 5.0$ and 10.0.

RESULTS AND DISCUSSION

Table 3 summarizes the experimental data that constitute conditions of optimum pipeline mixing at a T-junction. For a given jet-to-pipe diameter d/D, a limited range of velocity ratios R ($= u_o/v$) insure that the jet is centered on the pipe axis. The variation in optimum values of the velocity ratio R is < 10% from the average values computed from experimental data taken over the range $2.0 \leq x/D \leq 10$. This result is consistent with the view that pipe turbulence dominates the mixing process at distances greater than a Lagrangian length scale l_L from the point of injection. Since l_L/D

TABLE 3. SUMMARY OF EXPERIMENTAL DATA*

Run No.	d (cm)	d/D	x/D	u_o (cm/s)	R ($= u_o/v$)	Avg.	l_m/D	Avg.	$Re_j \times 10^{-3}$
					Re = 3.16×10^4				
A-1	0.1000	0.00875	2.0	6366	22.464	20.810	0.1965	0.1820	4.283
			3.0	6154	22.087		0.1932		4.140
			5.0	4881	19.670		0.1720		3.284
			10.0	4563	19.018		0.1664		3.069
A-2	0.1588	0.01389	2.0	3907	13.968	12.795	0.1940	0.1777	4.173
			3.0	3452	13.130		0.1824		3.687
			5.0	3031	12.303		0.1709		3.237
			10.0	2779	11.779		0.1639		2.968
A-3	0.2381	0.02083	2.0	2731	9.533	8.933	0.1986	0.1861	4.375
			3.0	2469	9.065		0.1888		3.956
			5.0	2319	8.786		0.1830		3.716
			10.0	2095	8.350		0.1739		3.357
A-4	0.3175	0.02778	2.0	2105	7.250	6.906	0.2014	0.1919	4.496
			3.0	1958	6.990		0.1942		4.182
			5.0	1863	6.820		0.1894		3.979
			10.0	1726	6.565		0.1824		3.687
A-5	0.4763	0.04167	2.0	1965	5.719	5.106	0.2383	0.2127	6.295
			3.0	1647	5.235		0.2181		5.276
			5.0	1385	4.800		0.2000		4.436
			10.0	1310	4.669		0.1946		4.197
A-6	0.6350	0.05556	2.0	1684	4.585	4.381	0.2547	0.2434	7.194
			3.0	1579	4.439		0.2466		6.706
			5.0	1476	4.289		0.2383		6.295
			10.0	1421	4.212		0.2340		6.070
					Re = 2.1×10^4				
B-1	0.1000	0.00875	2.0	4393	28.345	25.942	0.2480	0.2270	2.955
			3.0	3947	26.868		0.2351		2.655
			5.0	3438	25.076		0.2194		2.312
			10.0	3013	23.477		0.2054		2.027
B-2	0.1588	0.01389	2.0	2408	16.660	15.058	0.2314	0.2098	2.572
			3.0	2088	15.510		0.2154		2.230
			5.0	1819	14.480		0.2010		1.942
			10.0	1600	13.580		0.1886		1.709
B-3	0.2381	0.02083	2.0	1482	10.670	9.875	0.2222	0.2060	2.374
			3.0	1373	10.270		0.2140		2.200
			5.0	1124	9.240		0.1936		1.801
			10.0	1130	9.320		0.1940		1.811
B-4	0.3175	0.02778	2.0	1326	8.739	8.209	0.2427	0.2280	2.833
			3.0	1242	8.457		0.2349		2.653
			5.0	1116	8.015		0.2226		2.383
			10.0	1010	7.628		0.2119		2.158
B-5	0.4763	0.04167	2.0	1076	6.427	5.826	0.2678	0.2428	3.447
			3.0	936	5.993		0.2497		2.998
			5.0	795	5.525		0.2302		2.548
			10.0	748	5.360		0.2233		2.398
B-6	0.6350	0.05556	2.0	910	5.120	4.850	0.2844	0.2694	2.458
			3.0	842	4.922		0.2734		2.338
			5.0	784	4.751		0.2640		2.248
			10.0	737	4.606		0.2559		2.136
B-7	0.9525	0.08333	2.0	772	3.849	3.629	0.3208	0.3024	4.946
			3.0	655	3.546		0.2955		4.197
			5.0	690	3.639		0.3033		4.421
			10.0	632	3.482		0.2901		4.047
B-8	1.2700	0.11111	2.0	684	3.138	3.002	0.3487	0.3336	5.845
			3.0	638	3.031		0.3368		5.457
			5.0	605	2.952		0.3280		5.171
			10.0	579	2.887		0.3208		4.946
					Re = 1.26×10^4				
C-3	0.2381	0.02083	2.0	981	11.530	10.573	0.2404	0.2204	1.571
			3.0	879	10.930		0.2778		1.409
			5.0	767	10.210		0.2108		1.229
			10.0	681	9.623		0.2005		1.091
C-7	0.9525	0.08333	2.0	384	3.610	3.488	0.3009	0.2906	2.458
			3.0	365	3.521		0.2934		2.338
			5.0	351	3.453		0.2877		2.248
			10.0	333	3.366		0.2805		2.136

* $D = 11.43$ cm

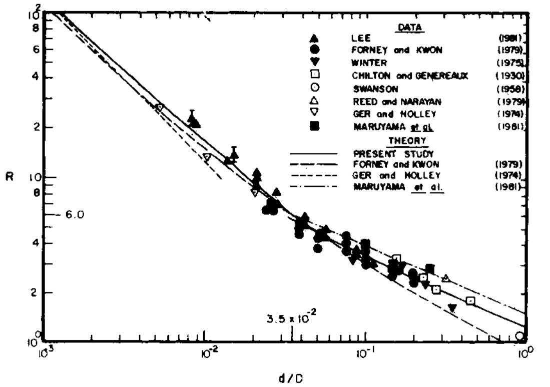

Figure 6. Velocity ratio R for optimum mixing conditions. Solid line for $R < 6$ is Eq. 13. Solid line for $R > 6$ is Eq. 14.

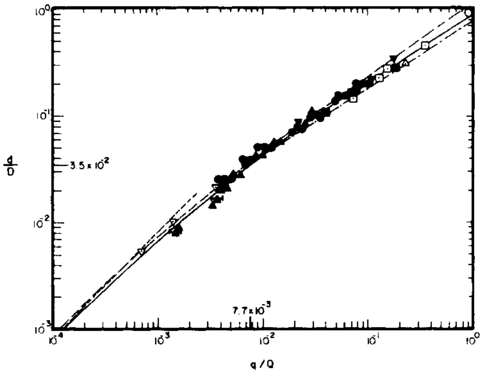

Figure 7. Ratio of flowrates $q/Q = (d/D)^2R$ for optimum mixing conditions. Solid line for $d/D > 0.035$ is derived from Eq. 13. Solid line for $d/D < 0.035$ is derived from Eq. 14. Symbols are the same as in Figure 6.

$= 0.4\,\mathrm{Re}^{1/8}$, the expected range in l_L for turbulent pipe flow is $1.28 < l_L/D < 2.28$ for a variation in pipe Reynolds numbers of $10^4 < \mathrm{Re} < 10^6$. The optimum velocity ratio R also indicates < 9% variation from the average computed value taken at fixed x/D for three pipe Reynolds numbers covering the range $1.2 \times 10^4 < \mathrm{Re} < 3.2 \times 10^4$.

Data representing the optimum dimensions for a T-junction from the experimentalists listed in Table 1 are plotted in Figure 6. Since not all of the data were taken with the same uniformity criterion, there is some expected scatter. The boundary conditions in Eqs. 7 and 8 that represent roughly the extent of jet induced turbulence and that correlate the data were found to be $x_f = 0.92$ in Eq. 7 and $x_g = 1.55$ in Eq. 8. Substituting for x_f and x_g, expressions are now derived for optimum mixing conditions which correlate the available data to within 20%. As plotted in Figure 6, the expressions are

$$\frac{d}{D} = \frac{0.27}{Rf^2}, \qquad R < 6 \tag{13}$$

and

$$\frac{d}{D} = \frac{0.28}{Rg^{3/2}}, \qquad R > 6. \tag{14}$$

Since the flow ratio is $q/Q = (d/D)^2R$, the data representing optimum flow ratios are replotted in Figure 7 as a function of d/D. One could, however, conveniently express q/Q in terms of R in the form

$$q/Q = \frac{0.073}{Rf^4}, \qquad R < 6 \tag{15}$$

and

$$q/Q = \frac{0.077}{Rg^3}, \qquad R > 6. \tag{16}$$

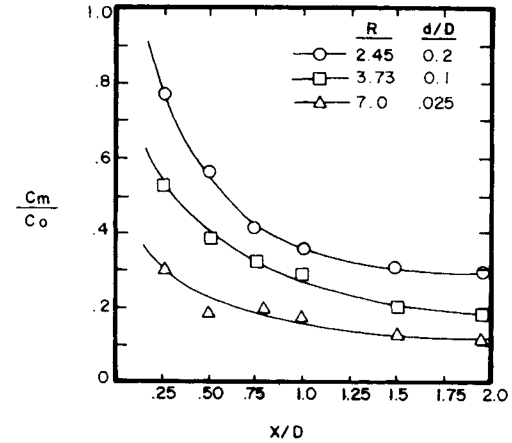

Figure 8. Concentration decay with distance from jet inlet (small R).

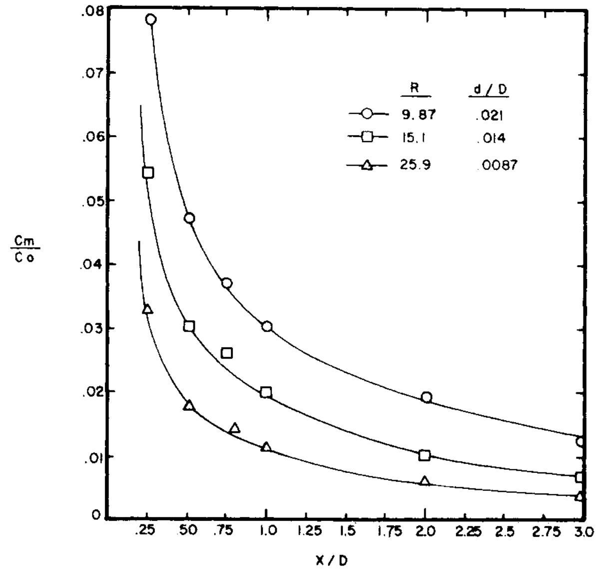

Figure 9. Concentration decay with distance from jet inlet (large R).

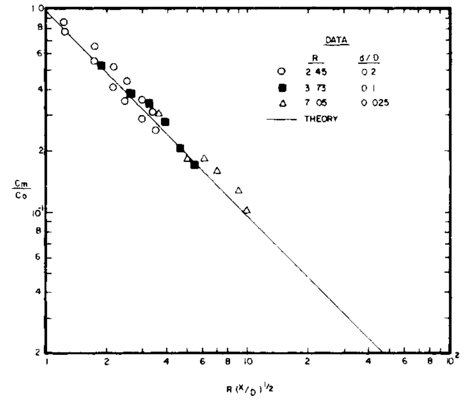

Figure 10. Concentration decay replotted from Figure 8. Solid line is Eq. 17.

The boundary conditions x_f, x_g introduced into Eqs. 7 and 8 leading to Eqs. 13 and 14 are roughly equal in magnitude to the Lagrangian length scale l_L on the pipe axis. This is consistent with the view that jet induced turbulence dominates mixing for $x < l_L$. The difference in the values chosen for the constants x_f and x_g may be attributable to the size of the potential core for the physically larger jets at smaller velocity ratios R or simply to the existance of wall effects. The values chosen for x_f and x_g may also be effected by the scatter in the data of Wright in Figure 2 representing the functions f and g or to the scatter in the data in Figure 6.

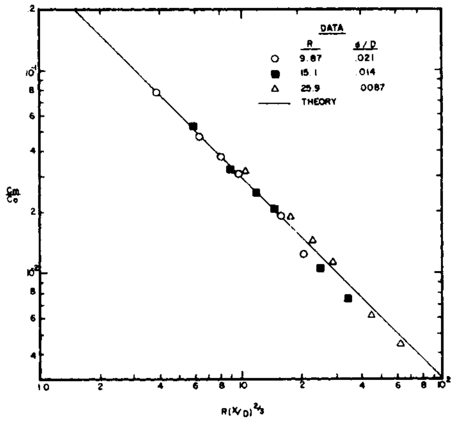

Figure 11. Concentration decay replotted from Figure 9. Solid line is Eq. 18.

Additional measurements were taken of the decay of the maximum methane concentration in the jet as a function of distance down the pipe axis from the point of injection. These data are shown in Figures 8 and 9 for both large and small velocity ratios. Replotting the data in Figure 10 and 11 as suggested earlier, we find that jet concentration decays in the following manner

$$\frac{c_m}{c_o} = \frac{1.0}{R(x/D)^{1/2}}, \quad R < 6 \tag{17}$$

and

$$\frac{c_m}{c_o} = \frac{0.32}{R(x/D)^{2/3}}, \quad R > 6 \tag{18}$$

with little scatter in the data.

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NOTATION

b = jet radius (cm)
b_o = jet inlet radius (cm)
c = mean jet tracer conc.
c_o = inlet tracer conc.
c_m = maximum jet tracer conc.
d = jet inlet diameter (cm)
D = pipe diameter (cm)
f = empirical function for near-field jet trajectory
g = empirical function for far-field jet trajectory
l_L = Lagrangian integral length scale, 0.4 $\mathrm{Re}^{1/8}$
l_m = jet momentum length, dR (cm)
q = jet inlet flowrate, $\pi/4\, d^2 u_o$ (cm^3/s)
Q = pipe flowrate, $\pi/4\, D^2 v$ (cm^3/s)
$(q/Q)_j$ = ratio of optimum flowrate for $\mathrm{Re}_j < 9 \times 10^3$
R = velocity ratio, u_o/v
R_j = optimum velocity ratio for $\mathrm{Re}_j < 9 \times 10^3$
Re = pipe Reynolds number, vD/ν
Re_j = jet Reynolds number, $u_o d/\nu$
u_o = mean jet inlet velocity (cm/s)
v = mean pipe velocity (cm/s)
x = distance down pipe axis from jet inlet (cm)
x_o = distance downstream from inlet dominated by jet mixing (cm)
x_c = intersection of near and far-field jet trajectories (cm)
x_f = boundary condition for pipeline jet trajectories with $R < 6$, x_o/D
x_g = boundary condition for pipeline jet trajectories with $R > 6$, x_o/D
z = distance along pipe diameter from jet inlet (cm)
z_c = intersection of near and far-field jet trajectories (cm)

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